

Q. 11. $\frac{dy}{dx} - y^2 \sin x = 0$

$$\Rightarrow \frac{dy}{dx} = y^2 \sin x$$

$$\Rightarrow \frac{1}{y^2} dy = \sin x dx$$

$$\Rightarrow \int_1^y \frac{1}{y^2} dy = \int_\pi^x \sin x dx$$

$$\Rightarrow \left[-\frac{1}{y} \right]_1^y = -\cos x \Big|_\pi^x$$

$$\Rightarrow \left[\frac{1}{y} \right]_1^y = \cos x \Big|_\pi^x$$

$$\Rightarrow \frac{1}{y} - 1 = \cos x - \cos \pi$$

$$\Rightarrow \frac{1}{y} - 1 = \cos x - (-1)$$

$$\Rightarrow \frac{1}{y} = \cos x + 2$$

$$\Rightarrow y = \frac{1}{\cos x + 2}$$

Q. 12. $xy \frac{dy}{dx} = y^2$

$$\Rightarrow \frac{1}{y} dy = \frac{1}{x} dx$$

$$\Rightarrow \int_2^y \frac{1}{y} dy = \int_1^x \frac{1}{x} dx$$

$$\Rightarrow \log_e y \Big|_2^y = \log_e x \Big|_1^x$$

$$\Rightarrow \log_e y - \log_e 2 = \log_e x - \log_e 1$$

$$\Rightarrow \log_e \frac{y}{2} = \log_e x$$

$$\Rightarrow \frac{y}{2} = x$$

$$\Rightarrow y = 2x$$

Q. 13. $v \frac{dv}{dx} = \cos^2 x$

$$\Rightarrow v dv = \cos^2 x dx$$

$$\Rightarrow \int_1^v v dv = \int_0^x \cos^2 x dx$$

$$\Rightarrow \frac{v^2}{2} \Big|_1^v = \frac{1}{2} \left(x + \frac{1}{2} \sin 2x \right) \Big|_0^x$$

$$\Rightarrow v^2 \Big|_1^v = \left(x + \frac{1}{2} \sin 2x \right) \Big|_0^x$$

$$\Rightarrow v^2 - 1 = x + \frac{1}{2} \sin 2x$$

$$\Rightarrow v^2 = x + \frac{1}{2} \sin 2x + 1$$

$$\Rightarrow v = \sqrt{x + \frac{1}{2} \sin 2x + 1}$$

Q. 14. $\frac{dy}{dx} = y \sin x$

$$\Rightarrow \frac{1}{y} dy = \sin x dx$$

$$\Rightarrow \int_{\sqrt{e}}^y \frac{1}{y} dy = \int_{\frac{\pi}{3}}^x \sin x dx$$

$$\Rightarrow \log_e y \Big|_{\sqrt{e}}^y = -\cos x \Big|_{\frac{\pi}{3}}^x$$

$$\Rightarrow \log_e y - \log_e \sqrt{e} = -\cos x + \cos \frac{\pi}{3}$$

$$\Rightarrow \log_e y - \frac{1}{2} = -\cos x + \frac{1}{2}$$

$$\Rightarrow \log_e y = 1 - \cos x$$

$$\Rightarrow y = e^{1 - \cos x}$$

Q. 15. (i) $\frac{dy}{dx} = y \cos x$

$$\Rightarrow \frac{1}{y} dy = \cos x dx$$

$$\Rightarrow \int_2^y \frac{1}{y} dy = \int_{\frac{\pi}{6}}^x \cos x dx$$

$$\Rightarrow \log_e y \Big|_2^y = \sin x \Big|_{\frac{\pi}{6}}^x$$

$$\Rightarrow \log_e y - \log_e 2 = \sin x - \sin \frac{\pi}{6}$$

$$\Rightarrow \log_e \frac{y}{2} = \sin x - \frac{1}{2}$$

$$\Rightarrow \frac{y}{2} = e^{\sin x - \frac{1}{2}}$$

$$\Rightarrow y = 2e^{\sin x - \frac{1}{2}}$$

(ii) Let $x = \frac{\pi}{2}$

$$\Rightarrow y = 2e^{1 - \frac{1}{2}} = 2e^{\frac{1}{2}} = 2\sqrt{e}$$

Q. 16. $2 \frac{dy}{dx} = xy + x$

$$\Rightarrow 2 \frac{dy}{dx} = x(y + 1)$$

$$\Rightarrow \frac{2}{y + 1} dy = x dx$$

$$\Rightarrow \int_2^y \frac{2}{y + 1} dy = \int_1^x x dx$$

$$\Rightarrow 2 \log_e |y + 1| \Big|_2^y = \frac{x^2}{2} \Big|_1^x$$

$$\Rightarrow 2(\log_e |y + 1| - \log_e 3) = \frac{x^2}{2} - \frac{1}{2}$$

$$\Rightarrow 2 \log_e \left| \frac{y + 1}{3} \right| = \frac{x^2 - 1}{2}$$

$$\Rightarrow \log_e \left| \frac{y + 1}{3} \right| = \frac{x^2 - 1}{4}$$

$$\Rightarrow \frac{y + 1}{3} = e^{\frac{x^2 - 1}{4}}$$