

$$\begin{aligned} \Rightarrow y + 1 &= 3e^{\frac{x^2-1}{4}} \\ \Rightarrow y &= 3e^{\frac{x^2-1}{4}} - 1 \quad \dots \text{let } x = 2 \\ \Rightarrow y &= 3e^{\frac{3}{4}} - 1 \\ \Rightarrow y &= 5.35 \end{aligned}$$

**Q. 17.**  $x \frac{dy}{dx} = \sqrt{4 - y^2}$

$$\begin{aligned} \Rightarrow \frac{1}{\sqrt{4 - y^2}} dy &= \frac{1}{x} dx \\ \Rightarrow \int_0^y \frac{1}{\sqrt{4 - y^2}} dy &= \int_1^x \frac{1}{x} dx \\ \Rightarrow \sin^{-1} \frac{y}{2} \Big|_0^y &= \log_e x \Big|_1^x \\ \Rightarrow \sin^{-1} \frac{y}{2} - \sin^{-1} 0 &= \log_e x - \log_e 1 \\ \Rightarrow \sin^{-1} \frac{y}{2} &= \log_e x \\ \Rightarrow \frac{y}{2} &= \sin(\log_e x) \\ \Rightarrow y &= 2 \sin(\log_e x) \\ &\dots \text{general solution} \end{aligned}$$

Let  $x = \sqrt[3]{e^{\frac{\pi}{6}}}$

$$\begin{aligned} \Rightarrow y &= 2 \sin \left( \log_e \sqrt[3]{e^{\frac{\pi}{6}}} \right) \\ \Rightarrow y &= 2 \sin \left( \frac{\pi}{6} \log_e e \right) \\ \Rightarrow y &= 2 \sin \frac{\pi}{6} \\ \Rightarrow y &= 2 \left( \frac{1}{2} \right) \\ \Rightarrow y &= 1 \end{aligned}$$

### Exercise 12F

**Q. 1.**  $\frac{d^2y}{dx^2} = 2 \frac{dy}{dx}$

$$\begin{aligned} \Rightarrow \frac{dy}{dx} &= 2v, \text{ where } v = \frac{dy}{dx} \\ \Rightarrow \int_1^v \frac{dv}{v} &= \int_0^x 2 dx \\ \Rightarrow \log_e v \Big|_1^v &= 2x \Big|_0^x \\ \Rightarrow \log_e v - \log_e 1 &= 2x - 2(0) \\ &\dots \log_e 1 = 0 \\ \Rightarrow \log_e v &= 2x \\ \Rightarrow v &= e^{2x} \quad \dots \text{End of Step 1} \end{aligned}$$

$$\begin{aligned} \Rightarrow \frac{dy}{dx} &= e^{2x} \\ \Rightarrow \int_0^y dy &= \int_0^x e^{2x} dx \\ \Rightarrow y \Big|_0^y &= \frac{1}{2} e^{2x} \Big|_0^x \\ \Rightarrow y - 0 &= \frac{1}{2} [e^{2x} - e^0] \\ \Rightarrow y &= \frac{1}{2} [e^{2x} - 1] \end{aligned}$$

**Q. 2.**  $\frac{d^2y}{(dx)^2} = v^2$ , where  $v = \frac{dx}{dt}$

$$\begin{aligned} \Rightarrow \int_1^v \frac{dv}{v^2} &= \int_0^t dt \\ \Rightarrow -\frac{1}{v} \Big|_1^v &= t \Big|_0^t \\ \Rightarrow -\frac{1}{v} - (-1) &= t - 0 \\ \Rightarrow 1 - \frac{1}{v} &= t \\ \Rightarrow \frac{1}{v} &= 1 - t \\ \Rightarrow v &= \frac{1}{1 - t} \quad \dots \text{End of Step 1} \\ \Rightarrow \frac{dx}{dt} &= \frac{1}{1 - t} \\ \Rightarrow \int_0^x dx &= \int_0^t \frac{dt}{1 - t} \\ \Rightarrow x \Big|_0^x &= -\log_e (1 - t) \Big|_0^t \\ \Rightarrow x - 0 &= -\log_e (1 - t) - (-\log_e 1) \\ &\dots \log_e 1 = 0 \\ \Rightarrow x &= \log_e (1 - t)^{-1} \\ \Rightarrow x &= \log_e \left( \frac{1}{1 - t} \right) \end{aligned}$$

**Q. 3.**  $\frac{d^2s}{dt^2} = 6$

$$\begin{aligned} \Rightarrow \frac{dv}{dt} &= 6, \text{ where } v = \frac{ds}{dt} \\ \Rightarrow \int_4^v dv &= \int_0^t 6 dt \\ \Rightarrow v \Big|_4^v &= 6t \Big|_0^t \\ \Rightarrow v - 4 &= 6t \\ \Rightarrow v &= 6t + 4 \quad \dots \text{End of Step 1} \\ \Rightarrow \frac{ds}{dt} &= 6t + 4 \\ \Rightarrow \int_0^s ds &= \int_0^t (6t + 4) dt \\ \Rightarrow s \Big|_0^s &= (3t^2 + 4t) \Big|_0^t \\ \Rightarrow s &= 3t^2 + 4t \end{aligned}$$