Q. 4. $\frac{d^2s}{dt^2} = -\left(\frac{ds}{dt}\right)^2$

$\Rightarrow \frac{dv}{dt} = -v^2$, where $v = \frac{ds}{dt}$

$\Rightarrow \int_{\frac{1}{2}}^{v} \frac{dv}{v^2} = -\int_0^t dt$

$\Rightarrow -\frac{1}{v}\Big|_{\frac{1}{2}}^{v} = -t\Big|_0^t$

$\Rightarrow -\frac{1}{v} - (-2) = -t$

$\Rightarrow 2 - \frac{1}{v} = -t$

$\Rightarrow \frac{1}{v} = t + 2$

$\Rightarrow v = \frac{1}{t+2}$ …End of Step 1

$\Rightarrow \frac{ds}{dt} = \frac{1}{t+2}$

$\Rightarrow \int_0^s ds = \int_0^t \frac{dt}{t+2}$

$\Rightarrow s\Big|_0^s = \log_e (t+2)\Big|_0^t$

$\Rightarrow s - 0 = \log_e (t+2) - \log_e 2$

$\Rightarrow s = \log_e \left(\frac{t+2}{2}\right)$

Q. 5. $\frac{d^2x}{dt^2} = \left(\frac{dx}{dt}\right)^2 + 1$

$\Rightarrow \frac{dv}{dt} = v^2 + 1$, where $v = \frac{dx}{dt}$

$\Rightarrow \int_0^v \frac{dv}{v^2+1} = \int_0^t dt$

$\Rightarrow \tan^{-1} v\Big|_0^v = t\Big|_0^t$

$\Rightarrow \tan^{-1} v - \tan^{-1} 0 = t - 0$ … $\tan^{-1} 0 = 0$

$\Rightarrow \tan^{-1} v = t$

$\Rightarrow v = \tan t$ …End of Step 1

$\Rightarrow \frac{dx}{dt} = \tan t$

$\Rightarrow \int_0^x dx = \int_0^t \tan t\, dt$

$\Rightarrow x\Big|_0^x = -\log_e |\cos t|\Big|_0^t$

$\Rightarrow x - 0 = -\log_e |\cos t| - (-\log_e |\cos 0|)$

…$\log_e |\cos 0| = \log_e 1 = 0$

$\Rightarrow x = -\log_e |\cos t|$

$\Rightarrow x = \log_e |\cos t|^{-1}$

$\Rightarrow x = \log_e \left|\frac{1}{\cos t}\right|$ **OR** $x = \log_e |\sec t|$

Q. 6. $\frac{d^2x}{dt^2} = 2x$ …let $v = \frac{dy}{dx} \Rightarrow \frac{dv}{dx} = \frac{d^2y}{dx^2}$

$\Rightarrow \frac{dv}{dx} = 2x$

$\Rightarrow \int_1^v dv = \int_0^x 2x\, dx$

$\Rightarrow v\Big|_1^v = x^2\Big|_0^x$

$\Rightarrow v - 1 = x^2 - 0$

$\Rightarrow v = x^2 + 1$ …End of Step 1

$\Rightarrow \frac{dy}{dx} = x^2 + 1$

$\Rightarrow \int_{10}^y dy = \int_0^x (x^2+1)\, dx$

$\Rightarrow y\Big|_{10}^y = \left(\frac{x^3}{3} + x\right)\Big|_0^x$

$\Rightarrow y - 10 = \frac{x^3}{3} + x$

$\Rightarrow y = \frac{x^3}{3} + x + 10$

Exercise 12G

Q. 1. $\frac{d^2y}{dx^2} = y$

$\Rightarrow v\frac{dv}{dy} = y$, where $v = \frac{dy}{dx}$

$\Rightarrow \int_1^v v\, dv = \int_1^y y\, dy$

$\Rightarrow \frac{v^2}{2}\Big|_0^v = \frac{y^2}{2}\Big|_0^y$

$\Rightarrow \frac{v^2}{2} = \frac{y^2}{2}$

$\Rightarrow v^2 = y^2$

$\Rightarrow v = y$

($v = -y$ won't work)

$\Rightarrow \frac{dy}{dx} = y$

$\Rightarrow \int_1^y \frac{dy}{y} = \int_0^x dx$

$\Rightarrow \log_e y\Big|_1^y = x\Big|_0^x$

$\Rightarrow \log_e y - \log_e 1 = x - 0$ …$\log_e 1 = 0$

$\Rightarrow \log_e y = x$

$\Rightarrow y = e^x$