Q. 2. $\frac{d^2s}{dt^2} = 9s$

$$\Rightarrow v\frac{dv}{ds} = 9s, \text{ where } v = \frac{ds}{dt}$$

$$\Rightarrow \int_{-6}^{v} v\,dv = \int_{2}^{s} 9s\,ds$$

$$\Rightarrow \frac{v^2}{2}\Big|_{-6}^{v} = \frac{9s^2}{2}\Big|_{2}^{s}$$

$$\Rightarrow \frac{v^2}{2} - 18 = \frac{9s^2}{2} - 18$$

$$\Rightarrow v^2 = 9s^2$$

$$\Rightarrow v = -3s$$

$(v = 3s \text{ won't work})$

$$\Rightarrow \frac{ds}{dt} = -3s$$

$$\Rightarrow \int_{e}^{s} \frac{ds}{s} = \int_{0}^{t} -3dt$$

$$\Rightarrow \log_e s\Big|_{e}^{s} = -3t\Big|_{0}^{t}$$

$$\Rightarrow \log_e s - \log_e e = -3t$$

$$\Rightarrow \log_e s - 1 = -3t$$

$$\Rightarrow \log_e s = 1 - 3t$$

$$\Rightarrow s = e^{1-3t}$$

Q. 3. $\frac{d^2y}{dx^2} = -\frac{2}{y^5}$

$$\Rightarrow v\frac{dv}{dy} = -\frac{2}{y^5}, \text{ where } v = \frac{dy}{dx}$$

$$\Rightarrow \int_{1}^{v} v\,dv = \int_{1}^{y} -\frac{2}{y^5}\,dy$$

$$\Rightarrow \frac{v^2}{2}\Big|_{1}^{v} = \frac{1}{2y^4}\Big|_{1}^{y}$$

$$\Rightarrow \frac{v^2}{2} - \frac{1}{2} = \frac{1}{2y^4} - \frac{1}{2}$$

$$\Rightarrow v^2 = \frac{1}{y^4}$$

$$\Rightarrow v = \frac{1}{y^2} \quad \left(v = -\frac{1}{y^2} \text{ won't work}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{y^2}$$

$$\Rightarrow \int_{1}^{y} y^2\,dy = \int_{\frac{1}{3}}^{x} dx$$

$$\Rightarrow \frac{y^3}{3}\Big|_{1}^{y} = x\Big|_{\frac{1}{3}}^{x}$$

$$\Rightarrow \frac{y^3}{3} - \frac{1}{3} = x - \frac{1}{3}$$

$$\Rightarrow y^3 = 3x$$

$$\Rightarrow y = \sqrt[3]{3x}$$

Q. 4. $\frac{d^2y}{dx^2} = -y$

$$\Rightarrow v\frac{dv}{dy} = -y, \text{ where } v = \frac{dy}{dx}$$

$$\Rightarrow \int_{2}^{v} v\,dv = \int_{0}^{y} -y\,dy$$

$$\Rightarrow \frac{v^2}{2}\Big|_{2}^{v} = -\frac{y^2}{2}\Big|_{0}^{y}$$

$$\Rightarrow \frac{v^2}{2} - 2 = -\frac{y^2}{2}$$

$$\Rightarrow v^2 - 4 = -y^2$$

$$\Rightarrow v^2 = 4 - y^2$$

$$\Rightarrow v = \sqrt{4 - y^2}$$

$(v = -\sqrt{4 - y^2} \text{ won't work})$

$$\Rightarrow \frac{dy}{dx} = \sqrt{4 - y^2}$$

$$\Rightarrow \int_{0}^{y} \frac{dy}{\sqrt{4 - y^2}} = \int_{0}^{x} dx$$

$$\Rightarrow \sin^{-1}\frac{y}{2}\Big|_{0}^{y} = x\Big|_{0}^{x}$$

$$\Rightarrow \sin^{-1}\frac{y}{2} - \sin^{-1} 0 = x - 0$$

$\ldots \sin^{-1} 0 = 0$

$$\Rightarrow \sin^{-1}\frac{y}{2} = x$$

$$\Rightarrow \frac{y}{2} = \sin x$$

$$\Rightarrow y = 2\sin x$$

Q. 5. $\frac{d^2x}{dt^2} = 2x(9 + x^2)$

$$\Rightarrow v\frac{dv}{dx} = 2x(9 + x^2), \text{ where } v = \frac{dx}{dt}$$

$$\Rightarrow \int_{9}^{v} v\,dv = \int_{0}^{x} 2x(9 + x^2)\,dx$$

$$\Rightarrow \frac{v^2}{2}\Big|_{9}^{v} = \frac{1}{2}(9 + x^2)^2\Big|_{0}^{x}$$

(using the substitution: $u = 9 + x^2$)

$$\Rightarrow \frac{v^2}{2} - \frac{81}{2} = \frac{1}{2}(9 + x^2)^2 - \frac{81}{2}$$

$$\Rightarrow v^2 = (9 + x^2)^2$$

$$\Rightarrow v = 9 + x^2$$

$(v = -(9 + x^2) \text{ won't work})$

$$\Rightarrow \frac{dx}{dt} = 9 + x^2$$

$$\Rightarrow \int_{3}^{x} \frac{dx}{9 + x^2} = \int_{0}^{t} dt$$

$$\Rightarrow \frac{1}{3}\tan^{-1}\frac{x}{3}\Big|_{3}^{x} = t\Big|_{0}^{t}$$