$$\Rightarrow \frac{1}{3}\left(\tan^{-1}\frac{x}{3} - \tan^{-1}1\right) = t - 0$$

$$\Rightarrow \frac{1}{3}\left(\tan^{-1}\frac{x}{3} - \frac{\pi}{4}\right) = t$$

$$\Rightarrow \tan^{-1}\frac{x}{3} - \frac{\pi}{4} = 3t$$

$$\Rightarrow \tan^{-1}\frac{x}{3} = 3t + \frac{\pi}{4}$$

$$\Rightarrow \frac{x}{3} = \tan\left(3t + \frac{\pi}{4}\right)$$

$$\Rightarrow x = 3\tan\left(3t + \frac{\pi}{4}\right)$$

Q. 6. $\dfrac{d^2x}{dt^2} = \dfrac{3x^2}{2}$

$$\Rightarrow v\frac{dv}{dx} = \frac{3x^2}{2}, \text{ where } v = \frac{dx}{dt}$$

$$\Rightarrow \int_{-8}^{v} 2v\,dv = \int_{4}^{x} 3x^2\,dx$$

$$\Rightarrow v^2\Big|_{-8}^{v} = x^3\Big|_{4}^{x}$$

$$\Rightarrow v^2 - 64 = x^3 - 64$$

$$\Rightarrow v^2 = x^3$$

$$\Rightarrow v = -\sqrt{x^3}$$

$(v = \sqrt{x^3}$ won't work)

$$\Rightarrow \frac{dx}{dt} = -x^{\frac{3}{2}}$$

$$\Rightarrow \int_{4}^{x} x^{-\frac{3}{2}}\,dx = \int_{0}^{t} -dt$$

$$\Rightarrow \frac{-2}{\sqrt{x}}\Big|_{4}^{x} = -t\Big|_{0}^{t}$$

$$\Rightarrow \frac{-2}{\sqrt{x}} - (-1) = -t$$

$$\Rightarrow \frac{2}{\sqrt{x}} = t + 1$$

$$\Rightarrow \frac{\sqrt{x}}{2} = \frac{1}{t+1}$$

$$\Rightarrow \sqrt{x} = \frac{2}{t+1}$$

$$\Rightarrow x = \left(\frac{2}{t+1}\right)^2$$

Q. 7. $\dfrac{d^2y}{dx^2} + \dfrac{2}{y^3} = 0$

$$\Rightarrow \frac{d^2y}{dx^2} = -2y^{-3}$$

Let $\dfrac{dy}{dx} = v \Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{dv}{dx} = \dfrac{dv}{dy}\cdot\dfrac{dy}{dx} = v\dfrac{dv}{dy}$

$$\Rightarrow v\frac{dv}{dy} = -2y^{-3}$$

$$\Rightarrow \int_{\sqrt{2}}^{v} v\,dv = \int_{1}^{y} -2y^{-3}\,dy$$

$$\Rightarrow \frac{v^2}{2}\Big|_{\sqrt{2}}^{v} = \frac{1}{y^2}\Big|_{1}^{y}$$

$$\Rightarrow \frac{v^2}{2} - 1 = \frac{1}{y^2} - 1$$

$$\Rightarrow v^2 = \frac{2}{y^2}$$

$$\Rightarrow v = \frac{\sqrt{2}}{y} \quad \left(v = -\frac{\sqrt{2}}{y} \text{ won't work}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{2}}{y}$$

$$\Rightarrow \int_{1}^{y} y\,dy = \int_{\sqrt{2}}^{\sqrt{18}} \sqrt{2}\,dx$$

$$\Rightarrow \frac{y^2}{2}\Big|_{1}^{y} = \sqrt{2}\,x\Big|_{\sqrt{2}}^{\sqrt{18}}$$

$$\Rightarrow \frac{y^2}{2} - \frac{1}{2} = 6 - 2$$

$$\Rightarrow y^2 - 1 = 8$$

$$\Rightarrow y^2 = 9$$

$$\Rightarrow y = \pm 3$$

Exercise 12H

Q. 1. (i)

$F = ma$

$$\Rightarrow -\frac{1}{3}mv = ma$$

$$\Rightarrow a = -\frac{1}{3}v$$

$$\Rightarrow \frac{dv}{dt} = -\frac{1}{3}v$$

$$\Rightarrow \int_{10}^{v} \frac{dv}{v} = \int_{0}^{t} -\frac{1}{3}\,dt$$

$$\Rightarrow \log_e v\Big|_{10}^{v} = -\frac{1}{3}t\Big|_{0}^{t}$$

$$\Rightarrow \log_e v - \log_e 10 = -\frac{1}{3}t$$

$$\Rightarrow \log_e \frac{v}{10} = -\frac{1}{3}t$$

$$\Rightarrow \frac{v}{10} = e^{-\frac{1}{3}t}$$

$$\Rightarrow v = 10e^{-\frac{1}{3}t}$$

…Equation 1

When $t = 3$, $v = 10e^{-1} = \dfrac{10}{e}$ m/s

(ii) $\dfrac{ds}{dt} = 10e^{-\frac{1}{3}t}$ …from Equation 1

$$\Rightarrow \int_{0}^{s} ds = \int_{0}^{t} 10e^{-\frac{1}{3}t}\,dt$$

$$\Rightarrow s\Big|_{0}^{s} = -30e^{-\frac{1}{3}t}\Big|_{0}^{t}$$