

$$\begin{aligned} \Rightarrow \frac{1}{3}(\tan^{-1} \frac{x}{3} - \tan^{-1} 1) &= t - 0 \\ \Rightarrow \frac{1}{3}(\tan^{-1} \frac{x}{3} - \frac{\pi}{4}) &= t \\ \Rightarrow \tan^{-1} \frac{x}{3} - \frac{\pi}{4} &= 3t \\ \Rightarrow \tan^{-1} \frac{x}{3} &= 3t + \frac{\pi}{4} \\ \Rightarrow \frac{x}{3} &= \tan(3t + \frac{\pi}{4}) \\ \Rightarrow x &= 3 \tan(3t + \frac{\pi}{4}) \end{aligned}$$

Q. 6. $\frac{d^2x}{dt^2} = \frac{3x^2}{2}$

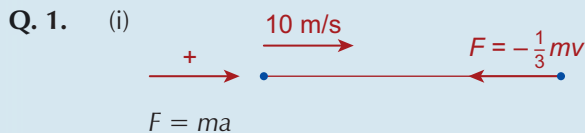
$$\begin{aligned} \Rightarrow v \frac{dv}{dx} &= \frac{3x^2}{2}, \text{ where } v = \frac{dx}{dt} \\ \Rightarrow \int_{-8}^v 2v \, dv &= \int_4^x 3x^2 \, dx \\ \Rightarrow v^2 \Big|_{-8}^v &= x^3 \Big|_4^x \\ \Rightarrow v^2 - 64 &= x^3 - 64 \\ \Rightarrow v^2 &= x^3 \\ \Rightarrow v &= -\sqrt{x^3} \\ &\quad (v = \sqrt{x^3} \text{ won't work}) \\ \Rightarrow \frac{dx}{dt} &= -x^{\frac{3}{2}} \\ \Rightarrow \int_4^x x^{-\frac{3}{2}} \, dx &= \int_0^t -dt \\ \Rightarrow \frac{-2}{\sqrt{x}} \Big|_4^x &= -t \Big|_0^t \\ \Rightarrow \frac{-2}{\sqrt{x}} - (-1) &= -t \\ \Rightarrow \frac{2}{\sqrt{x}} &= t + 1 \\ \Rightarrow \frac{\sqrt{x}}{2} &= \frac{1}{t + 1} \\ \Rightarrow \sqrt{x} &= \frac{2}{t + 1} \\ \Rightarrow x &= \left(\frac{2}{t + 1}\right)^2 \end{aligned}$$

Q. 7. $\frac{d^2y}{dx^2} + \frac{2}{y^3} = 0$

$$\begin{aligned} \Rightarrow \frac{d^2y}{dx^2} &= -2y^{-3} \\ \text{Let } \frac{dy}{dx} = v &\Rightarrow \frac{d^2y}{dx^2} = \frac{dv}{dx} = \frac{dv}{dy} \cdot \frac{dy}{dx} = v \frac{dv}{dy} \\ \Rightarrow v \frac{dv}{dy} &= -2y^{-3} \\ \Rightarrow \int_{\sqrt{2}}^v v \, dv &= \int_1^y -2y^{-3} \, dy \\ \Rightarrow \frac{v^2}{2} \Big|_{\sqrt{2}}^v &= \frac{1}{y^2} \Big|_1^y \end{aligned}$$

$$\begin{aligned} \Rightarrow \frac{v^2}{2} - 1 &= \frac{1}{y^2} - 1 \\ \Rightarrow v^2 &= \frac{2}{y^2} \\ \Rightarrow v &= \frac{\sqrt{2}}{y} \quad (v = -\frac{\sqrt{2}}{y} \text{ won't work}) \\ \Rightarrow \frac{dy}{dx} &= \frac{\sqrt{2}}{y} \\ \Rightarrow \int_1^y y \, dy &= \int_{\sqrt{2}}^{\sqrt{18}} \sqrt{2} \, dx \\ \Rightarrow \frac{y^2}{2} \Big|_1^y &= \sqrt{2} x \Big|_{\sqrt{2}}^{\sqrt{18}} \\ \Rightarrow \frac{y^2}{2} - \frac{1}{2} &= 6 - 2 \\ \Rightarrow y^2 - 1 &= 8 \\ \Rightarrow y^2 &= 9 \\ \Rightarrow y &= \pm 3 \end{aligned}$$

Exercise 12H



$$\begin{aligned} \Rightarrow -\frac{1}{3}mv &= ma \\ \Rightarrow a &= -\frac{1}{3}v \\ \Rightarrow \frac{dv}{dt} &= -\frac{1}{3}v \\ \Rightarrow \int_{10}^v \frac{dv}{v} &= \int_0^t -\frac{1}{3} \, dt \\ \Rightarrow \log_e v \Big|_{10}^v &= -\frac{1}{3}t \Big|_0^t \\ \Rightarrow \log_e v - \log_e 10 &= -\frac{1}{3}t \\ \Rightarrow \log_e \frac{v}{10} &= -\frac{1}{3}t \\ \Rightarrow \frac{v}{10} &= e^{-\frac{1}{3}t} \\ \Rightarrow v &= 10e^{-\frac{1}{3}t} \end{aligned}$$

...Equation 1

When $t = 3$, $v = 10e^{-1} = \frac{10}{e}$ m/s

(ii) $\frac{ds}{dt} = 10e^{-\frac{1}{3}t}$...from Equation 1

$$\begin{aligned} \Rightarrow \int_0^s ds &= \int_0^t 10e^{-\frac{1}{3}t} \, dt \\ \Rightarrow s \Big|_0^s &= -30e^{-\frac{1}{3}t} \Big|_0^t \end{aligned}$$