

$$\Rightarrow s - 0 = -30e^{-\frac{1}{3}t} - (-30)$$

$$\Rightarrow s = 30\left(1 - e^{-\frac{1}{3}t}\right)$$

When $t = 3$

$$s = 30(1 - e^{-1}) = 30\left(1 - \frac{1}{e}\right)$$

(iii) As $t \rightarrow \infty$, $e^{-\frac{1}{3}t} \rightarrow 0$

$$\therefore s \rightarrow 30(1 - 0) = 30 \text{ m}$$

Q. 2. $a = 25v + v^3$

$$\Rightarrow \frac{v dv}{ds} = v(25 + v^2)$$

$$\Rightarrow dv = (25 + v^2) ds \quad \text{QED}$$

$$\Rightarrow \int_0^v \frac{dv}{25 + v^2} = \int_0^{0.01} ds$$

$$\Rightarrow \frac{1}{5} \left[\tan^{-1} \frac{v}{5} \right]_0^v = [s]_0^{0.01}$$

$$\Rightarrow \tan^{-1} \frac{v}{5} = 0.05$$

$$\Rightarrow \frac{v}{5} = \tan 0.05$$

$$\Rightarrow v = 5 \tan 0.05$$

$$\Rightarrow v = 0.25 \text{ m/s}$$

Q. 3. $a = v^2 + 100$

$$\Rightarrow \frac{dv}{dt} = v^2 + 100$$

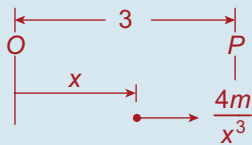
$$\Rightarrow \int_0^{20} \frac{dv}{v^2 + 100} = \int_0^t dt$$

$$\Rightarrow \frac{1}{10} \left[\tan^{-1} \frac{v}{10} \right]_0^{20} = t$$

$$\Rightarrow t = \frac{1}{10} [\tan^{-1} 2]$$

$$\Rightarrow t = 0.11 \text{ s}$$

Q. 4. **Forces**



NZL: $\Sigma F = ma$

$$\rightarrow \frac{4m}{x^3} = m \frac{dv}{dx}$$

(i) $\Rightarrow v dv = 4x^{-3} dx \quad \text{QED}$

$$\Rightarrow \int_{-\frac{\sqrt{5}}{3}}^v v dv = 4 \int_3^x x^{-3} dx$$

$$\Rightarrow \frac{1}{2} \left[v^2 \right]_{-\frac{\sqrt{5}}{3}}^v = -2 \left[\frac{1}{x^2} \right]_3^x$$

$$\Rightarrow v^2 - \frac{5}{9} = 4 \left[\frac{1}{9} - \frac{1}{x^2} \right]$$

$$\Rightarrow v^2 = 1 - \frac{4}{x^2}$$

$$\Rightarrow v = \sqrt{1 - \frac{4}{x^2}}$$

(ii) For $v = 0$, $\frac{4}{x^2} = 1 \Rightarrow x = 2 \text{ m}$

(iii) When $x = \frac{5}{2}$

$$v = \sqrt{1 - \frac{4}{\left(\frac{25}{4}\right)}}$$

$$= \sqrt{1 - \frac{16}{25}}$$

$$= \sqrt{\frac{9}{25}}$$

$$\Rightarrow v = \frac{3}{5} \text{ m/s}$$

Q. 5. (i) $F = ma$

$$\Rightarrow 8a = (40 - 3\sqrt{x})$$

$$\Rightarrow a = \frac{40 - 3\sqrt{x}}{8}$$

$$\Rightarrow v \frac{dv}{dx} = \frac{40 - 3\sqrt{x}}{8}$$

$$\Rightarrow \int_0^v 8v dv = \int_0^x (40 - 3\sqrt{x}) dx$$

$$\Rightarrow 4v^2 \Big|_0^v = (40x - 2\sqrt{x^3}) \Big|_0^x$$

$$\Rightarrow 4v^2 = 40x - 2\sqrt{x^3}$$

$$\Rightarrow v^2 = 10x - \frac{1}{2}\sqrt{x^3}$$

$$\Rightarrow v = \sqrt{10x - \frac{1}{2}\sqrt{x^3}} \quad \dots \text{let } x = 100$$

$$\Rightarrow v = \sqrt{1000 - \frac{1}{2}(1000)}$$

$$= \sqrt{500}$$

$$= 22.36 \text{ m/s}$$

(ii) $v^2 = 10x - \frac{1}{2}\sqrt{x^3} \quad \dots \text{let } v = 0$

$$\Rightarrow 10x - \frac{1}{2}\sqrt{x^3} = 0$$

$$\Rightarrow 20x - \sqrt{x^3} = 0$$

$$\Rightarrow 20x - x\sqrt{x} = 0$$

$$\Rightarrow x(20 - \sqrt{x}) = 0$$

$$\Rightarrow 20 - \sqrt{x} = 0$$

$$\Rightarrow \sqrt{x} = 20$$

$$\Rightarrow x = 400 \text{ m}$$