(i) When $x = \sqrt{3}d$,

$$v = \sqrt{\frac{1}{d^4} - \frac{1}{9d^4}}$$

$$\Rightarrow v = \sqrt{\frac{9-1}{9d^4}}$$

$$\Rightarrow v = \frac{2\sqrt{2}}{3d^2} \quad \textbf{QED}$$

(ii) From ①, $v_T = \underset{x\to\infty}{\sin} \sqrt{\frac{1}{d^4} - \frac{1}{x^4}}$

where v_T = Terminal Velocity

$$\Rightarrow v_T = \frac{1}{d^2} \quad \textbf{QED}$$

Q. 10. $\frac{d^2s}{dt^2} = a$, Let $\frac{ds}{dt} = v$

$$\Rightarrow \frac{dv}{dt} = a$$

$$\Rightarrow \int_u^v dv = a\int_0^t dt$$

$$\Rightarrow v - u = at$$

$$\Rightarrow v = u + at$$

$$\Rightarrow \frac{ds}{dt} = u + at$$

$$\Rightarrow \int_0^s ds = \int_0^t (u + at)\, dt$$

$$\Rightarrow s = ut + \frac{at^2}{2} \quad \textbf{QED}$$

Q. 11. $\frac{d^2s}{dt^2} = a$

$$\Rightarrow v\frac{dv}{ds} = a$$

$$\Rightarrow \int_u^v v\, dv = a\int_0^s ds$$

$$\Rightarrow \frac{1}{2}\left[v^2\right]_u^v = as$$

$$\Rightarrow v^2 - u^2 = 2as$$

$$\Rightarrow v^2 = u^2 + 2as \quad \textbf{QED}$$

Exercise 12I

Q. 1. $\frac{dv}{dt} = v^2 + 4$

Note: the integration does require a substitution.

$$\Rightarrow \int_2^v \frac{dv}{v^2 + 4} = \int_0^t dt$$

$$\Rightarrow \frac{1}{2}\tan^{-1}\frac{v}{2}\Big|_2^v = t\Big|_0^t$$

$$\Rightarrow \frac{1}{2}\left[\tan^{-1}\frac{v}{2} - \tan^{-1}1\right] = t - 0$$

$$\ldots\tan^{-1}1 = \frac{\pi}{4}$$

$$\Rightarrow t = \frac{1}{2}\left[\tan^{-1}\frac{v}{2} - \frac{\pi}{4}\right] \qquad \ldots\text{let } v = 6$$

$$\Rightarrow t = \frac{1}{2}\left[\tan^{-1}3 - \frac{\pi}{4}\right]$$

$$= 0.23 \text{ s}$$

Q. 2. $\frac{d^2s}{dt^2} = -\left(\frac{ds}{dt}\right)^2$

$$\Rightarrow \frac{dv}{dt} = -v^2 \qquad \text{where } v = \frac{ds}{dt}$$

$$\Rightarrow \int_1^v \frac{dv}{v^2} = -\int_0^t dt$$

$$\Rightarrow -\frac{1}{v}\Big|_1^v = -t\Big|_0^t$$

$$\Rightarrow \frac{1}{v}\Big|_1^v = t\Big|_0^t$$

$$\Rightarrow \frac{1}{v} - 1 = t - 0$$

$$\Rightarrow \frac{1}{v} = t + 1$$

$$\Rightarrow v = \frac{1}{t+1}$$

$$\Rightarrow \frac{ds}{dt} = \frac{1}{t+1}$$

$$\Rightarrow \int_0^s ds = \int_0^1 \frac{dt}{t+1}$$

$$\Rightarrow s\Big|_0^s = \log_e(t+1)\Big|_0^1$$

$$\Rightarrow s - 0 = \log_e 2 - \log_e 1 \qquad \ldots\log_e 1 = 0$$

$$\Rightarrow s = \log_e 2$$

Q. 3. $\frac{d^2s}{dt^2} = \frac{1}{2}\left(\frac{ds}{dt}\right)$

$$\Rightarrow \frac{dv}{dt} = \frac{1}{2}v \qquad \text{where } v = \frac{ds}{dt}$$

$$\Rightarrow \int_3^v \frac{dv}{v} = \int_0^t \frac{1}{2}\, dt$$

$$\Rightarrow \log_e v\Big|_3^v = \frac{1}{2}t\Big|_0^t$$

$$\Rightarrow \log_e v - \log_e 3 = \frac{1}{2}t$$