

$$\Rightarrow \log_e \frac{v}{3} = \frac{1}{2}t$$

$$\Rightarrow \frac{v}{3} = e^{\frac{1}{2}t}$$

$$\Rightarrow v = 3e^{\frac{1}{2}t}$$

$$\Rightarrow \frac{ds}{dt} = 3e^{\frac{1}{2}t}$$

$$\Rightarrow \int_0^s ds = \int_4^5 3e^{\frac{1}{2}t} dt$$

(since the 5<sup>th</sup> second is from  $t = 4$  to  $t = 5$ )

$$\Rightarrow s \Big|_0^s = 6e^{\frac{1}{2}t} \Big|_4^5$$

$$\Rightarrow s = 6(e^{\frac{5}{2}} - e^2) \text{ m}$$

**Q. 4.**  $\frac{d^2s}{dt^2} = k\left(\frac{ds}{dt}\right)^2 \Rightarrow \frac{dv}{dt} = -kv^2$

when  $t = 0, s = 0, v = 20$ ;

when  $s = 100, v = 10$ .

$$\Rightarrow \int_{20}^v v^{-2} dv = \int_0^t -k dt$$

$$\Rightarrow -\frac{1}{v} \Big|_{20}^v = -kt \Big|_0^t$$

$$\Rightarrow \frac{1}{v} \Big|_{20}^v = kt \Big|_0^t$$

$$\Rightarrow \frac{1}{v} - \frac{1}{20} = kt$$

$$\Rightarrow \frac{1}{v} = kt + \frac{1}{20}$$

$$\Rightarrow \frac{1}{v} = \frac{20kt + 1}{20}$$

$$\Rightarrow v = \frac{20}{20kt + 1}$$

$$\Rightarrow \frac{ds}{dt} = \frac{20}{20kt + 1}$$

$$\Rightarrow \int_0^{100} ds = \int_0^t \frac{20}{20kt + 1} dt$$

$$\Rightarrow s \Big|_0^{100} = \frac{1}{k} \log_e (20kt + 1) \Big|_0^t$$

$$\Rightarrow \frac{1}{k} [\log_e (20kt + 1) - \log_e 1] = 100 - 0$$

... $\log_e 1 = 0$

$$\Rightarrow \frac{1}{k} \log_e (20kt + 1) = 100$$

Equation 1

...when  $\frac{20}{20kt + 1} = 10$

Equation 2

$$\Rightarrow 20kt + 1 = 2$$

Equation 3

$$\Rightarrow \frac{1}{k} \log_e 2 = 100$$

$$\Rightarrow k = \frac{1}{100} \log_e 2$$

Putting this into equation 3 gives

$$\frac{1}{5} (\log_e 2)t + 1 = 2$$

$$\Rightarrow t = \frac{5}{\log_e 2} = 7.2 \text{ s}$$

**Q. 5.**  $a = -\frac{v^3}{25} \dots \textcircled{1}$

$$\Rightarrow v \frac{dv}{dx} = -\frac{v^3}{25}$$

$$\Rightarrow \int_{25}^v \frac{dv}{v^2} = -\frac{1}{25} \int_0^x dx$$

$$\Rightarrow -\left[\frac{1}{v}\right]_{25}^v = -\frac{1}{25} [x]_0^x$$

$$\Rightarrow \frac{1}{v} - \frac{1}{25} = \frac{x}{25}$$

$$\Rightarrow \frac{1}{v} = \frac{x+1}{25}$$

(i)  $\Rightarrow v = \frac{25}{x+1}$

$$\Rightarrow v \Big|_{x=99} = \frac{1}{4} \text{ m/s}$$

(ii)  $\frac{dv}{dt} = -\frac{v^3}{25}$  from  $\textcircled{1}$

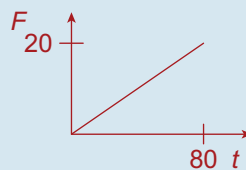
$$\Rightarrow \int_0^5 v^{-3} dv = -\frac{1}{25} \int_0^t dt$$

$$\Rightarrow -\frac{1}{2} \left[\frac{1}{v^2}\right]_0^5 = -\frac{1}{25} t$$

$$\Rightarrow t = \frac{25}{2} \left[\frac{1}{25} - \frac{1}{100}\right]$$

$$\Rightarrow t = \frac{3}{8} \text{ s}$$

**Q. 6.**



$$F = \frac{20}{80}t + 0$$

$$\Rightarrow F = \frac{t}{4}$$

$$\boxed{2} \rightarrow \frac{t}{4}$$

NZL:  $\Sigma F = ma$

$$+ \frac{t}{4} = 2a$$

(i)  $\Rightarrow a = \frac{t}{8}$  **QED**

(ii)  $\frac{dv}{dt} = \frac{t}{8}$

$$\Rightarrow \int_0^v dv = \frac{1}{8} \int_0^t t dt$$

$$\Rightarrow v = \frac{t^2}{16} \dots \textcircled{1}$$