$F_{left} = F_{right} \Rightarrow F_1 \sin\theta = F_2 \sin\theta \Rightarrow F_1 = F_2$

$\Rightarrow k_1(l - l_1) = k_2(l - l_2)$

$\Rightarrow k_1 l - k_1 l_1 = k_2 l - k_2 l_2$

$\Rightarrow l(k_1 - k_2) = k_1 l_1 - k_2 l_2$

$$\Rightarrow l = \frac{k_1 l_1 - k_2 l_2}{k_1 - k_2}$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{d}{l} = \frac{d(k_1 - k_2)}{k_1 l_1 - k_2 l_2}$$ **QED**

Exercise 13B

Q. 1. (i) $\frac{2\pi}{7} = \frac{2\pi}{\omega} \Rightarrow \omega = 7$

Max. velocity $= \omega A = 7(5) = 35$ m/s

(ii) Max. acceleration $= \omega^2 A = (7)^2(5)$

$= 245$ m/s^2

(iii) Total distance covered $= 4A = 20$ m

Time taken $= \frac{2\pi}{7}$

Average speed $= \frac{\text{Distance}}{\text{Time}}$

$= \frac{20}{\frac{2\pi}{7}}$

$= \frac{140}{2\pi}$

$= \frac{70}{\pi}$ m/s

Q. 2. $v^2 = \omega^2(A^2 - x^2)$

$x = \sqrt{7}, v = 9$

$\Rightarrow 81 = \omega^2(A^2 - 7)$ ..... **Equation 1**

$x = 2, v = 6\sqrt{3}$

$\Rightarrow 108 = \omega^2(A^2 - 4)$ ..... **Equation 2**

Dividing equation **1** by **2** gives

$$\frac{81}{108} = \frac{\omega^2(A^2 - 7)}{\omega^2(A^2 - 4)} \Rightarrow \frac{A^2 - 7}{A^2 - 4} = \frac{3}{4}$$

$\Rightarrow 3A^2 - 12 = 4A^2 - 28 \Rightarrow A = 4$

Putting this result into equation **1** gives:

$81 = \omega^2(4^2 - 7) \Rightarrow \omega^2 = 9 \Rightarrow \omega = 3$

Periodic time $T = \frac{2\pi}{\omega} = \frac{2\pi}{3}$ s

Q. 3. Max. velocity $= \omega A = 6$ .... **Equation 1**

Max. acceleration $= \omega^2 A$

$= 12$ .... **Equation 2**

Dividing equation **2** by equation **1** gives $\omega = 2$

Therefore, $A = 3$

Periodic time $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$ s

To find a when $v = 2\sqrt{5}$:

Step 1: Find x when $v = 2\sqrt{5}$:

$v^2 = \omega^2(A^2 - x^2)$

$\Rightarrow 20 = 2^2(3^2 - x^2)$

$\Rightarrow x = \pm 2$

Step 2: Find a when $x = \pm 2$

$a = -\omega^2 x = -(2)^2(\pm 2) = \pm 8$ m/s^2

The magnitude of the acceleration is 8 m/s^2

Q. 4. (i) $v^2 = \omega^2(A^2 - x^2)$

$v = 8$ when $x = 1$

$\Rightarrow 64 = \omega^2(A^2 - 1)$ .... **Equation 1**

$v = 4$ when $x = 7$

$\Rightarrow 16 = \omega^2(A^2 - 49)$ .... **Equation 2**

Dividing **1** by **2** gives

$$\frac{64}{16} = \frac{\omega^2(A^2 - 1)}{\omega^2(A^2 - 49)}$$

$$\Rightarrow \frac{A^2 - 1}{A^2 - 49} = \frac{4}{1}$$

$\Rightarrow 4A^2 - 196 = A^2 - 1$

$\Rightarrow A = \sqrt{65}$

(ii) Putting this result into equation **1** gives

$64 = \omega^2(65 - 1)$

$\Rightarrow \omega = 1$

$\therefore T = \frac{2\pi}{\omega} = \frac{2\pi}{1} = 2\pi$ s

(iii) When $x = 0$, $v^2 = \omega^2(A^2 - x^2)$

$= 1(65 - 0) = 65$

$\therefore v = \sqrt{65}$ m/s

Q. 5. When $x = 1$, $v = 3$, $a = 3$

So $v = \omega\sqrt{A^2 - x^2}$ ...(1)

and $a = \omega^2 x$ ...(2)

$\Rightarrow 3 = \omega^2(1)$

$\Rightarrow \omega = \sqrt{3}$

$\therefore$ From (1), $3 = \sqrt{3}\sqrt{A^2 - 1}$

$\Rightarrow A = 2$

From (2), $a_{MAX} = \omega^2 A$

$\Rightarrow a_{MAX} = 3(2)$

$\Rightarrow a_{MAX} = 6$ m/s^2