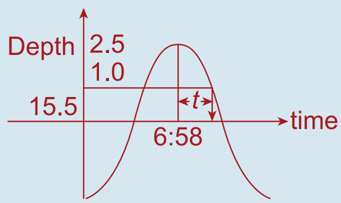
Amplitude:

$A = \frac{18 - 13}{2} \Rightarrow A = 2.5$ m

$\therefore$ Mean level (Equilibrium Position) $= 15.5$ m



6:58 + t is latest time ship can leave.

$x = A \cos \omega t$

$1 = 2.5 \cos \frac{\pi t}{6}$

$\Rightarrow t = 2.214$ hrs

$\Rightarrow t = 2$ hrs $12' \, 50.62''$

so $6{:}58 + 2{:}12' \, 50.62''$

$= 9{:}10' \, 50.62''$ **OR** 9:10 PM

Exercise 13C

Q. 1. $x = 3 \sin 5t. \therefore \frac{dx}{dt} = 15 \cos 5t.$

$\therefore \frac{d^2x}{dt^2} = -75 \sin 5t$

$= -25(3 \sin 5t) = -25x$

Since the acceleration is proportional to x but in the opposite direction, it will perform SHM.

$x = 3 \sin 5t \Rightarrow 1.5 = 3 \sin 5t$

$\Rightarrow \sin 5t = \frac{1}{2} \Rightarrow 5t, \frac{\pi}{6} \Rightarrow t = \frac{\pi}{30}$ s

Q. 2. (i) $x = 4 \cos 2t. \therefore \frac{dx}{dt} = -8 \sin 2t$

$\therefore \frac{d^2x}{dt^2} = -16 \cos 2t = -4(4 \cos 2t)$

$= -4x$

Since the acceleration is proportional to the distance from p, but in the opposite direction, it will perform SHM ($A = 4$, $\omega = 2$)

(ii) Greatest distance $= A = 4$ m

(iii) Its velocity is zero at the extreme point. Since the clock starts when the particle is at an extreme point, use $x = 4 \cos 2t$.

$x = 4 \cos 2t \Rightarrow 2.5 = 4 \cos 2t$

$\Rightarrow \cos 2t = 0.625$

$\Rightarrow 2t = \cos^{-1}(0.625) \Rightarrow 2t = 0.8956$

$\Rightarrow t = 0.4478$ s

Q. 3. $x = 9 \cos 3t$

(i) For $t = 0$, $x = 9 \cos 0 \Rightarrow x = 9$

(ii) **Note:** x is measured from equilibrium.

So, when particle has travelled 2 metres, $x = 7$

$x = 9 \cos 3t$

$\Rightarrow 7 = 9 \cos 3t$

$\Rightarrow \cos 3t = \frac{7}{9}$

$\Rightarrow 3t = \cos^{-1} \frac{7}{9}$

$\Rightarrow t = 0.227$ s

Q. 4. (i) $x = 13 \sin(\omega t + \varepsilon)$,

when $t = 0$, $x = 5 \Rightarrow 5 = 13 \sin \varepsilon$

$\sin \varepsilon = \frac{5}{13} = 0.3846$

$\Rightarrow \varepsilon = \sin^{-1}(0.3846) = 0.3948$

(ii) $v^2 = \omega^2(A^2 - x^2)$, $v = 24$ when $x = 5$. Also $A = 13$

$\therefore (24)^2 = \omega^2(13^2 - 5^2)$

$\Rightarrow 576 = \omega^2(144) \Rightarrow \omega = 2$

(iii) $x = 0 \Rightarrow 13 \sin(\omega t + \varepsilon) = 0$

$\Rightarrow \omega t + \varepsilon = 0$ **OR** π **OR** 2π etc.

$\Rightarrow 2t + 0.3948$

$= 0$ **OR** 3.1416 **OR** 6.2832 etc.

The first time ($t > 0$) will be when $2t + 0.3948 = 3.1416$

$\Rightarrow 2t = 2.7468$

$\Rightarrow t = 1.3734$ s

Q. 5. (i) $x = 3 \cos 2t + 4 \sin 2t$

$A = \sqrt{3^2 + 4^2} = 5$

$T = \frac{2\pi}{2} = \pi$

(ii) $x = 8 \cos 4t + 6 \sin 4t$

$A = \sqrt{8^2 + 6^2} = 10$

$T = \frac{2\pi}{4} = \frac{\pi}{2}$