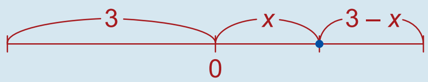
Exercise 13D

Q. 1. (i)



$F_r = k(l - l_0)$
$= 2(3 - x - 1)$
$= 4 - 2x$

$F_l = k(l - l_0)$
$= 2(3 + x - 1)$
$= 4 + 2x$

$F = F_r - F_l$
$= 4 - 2x - 4 - 2x$
$= -4x$

$F = ma \Rightarrow -4x = 1(a) \Rightarrow a = -4x$

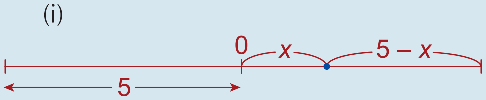
This is SHM with $\omega = 2$.

(ii) Periodic time $= \dfrac{2\pi}{2} = \pi$ s

(iii) A = original distance from 0 = 1.

Midway between the walls $\Rightarrow x = 0$
$\Rightarrow v^2 = \omega^2(A^2 - x^2) \Rightarrow v^2 = 4(1 - 0)$
$\Rightarrow v = 2$ m/s

Q. 2. (i)



$F_r = k(l - l_0)$
$= 9(5 - x - 1)$
$= 36 - 9x$

$F_l = 9(5 + x - 1)$
$= 36 + 9x$

$F_t = F_r - F_l$
$= 36 - 9x - 36 - 9x$
$= -18x$

$F = ma$

$\Rightarrow -18x = \dfrac{1}{2}a$

$\Rightarrow a = -36x$

It will perform SHM with $\omega = 6$. When it is released, $x = 2$. Therefore $A = 2$.

(ii) It starts from an extreme point.
$\therefore x = a \cos \omega t$ i.e. $x = 2 \cos 6t$.

We want to find t when $x = 1$.

$\therefore 1 = 2 \cos 6t$

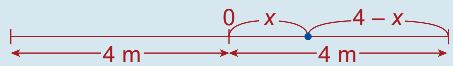
$\Rightarrow \cos 6t = \dfrac{1}{2}$

$\Rightarrow 6t = \dfrac{\pi}{3}$

$\Rightarrow t = \dfrac{\pi}{18}$ s

(iii) $v^2 = \omega^2(A^2 - x^2)$
$\Rightarrow v^2 = 6^2(2^2 - 1^2) = 108$
$\Rightarrow v = \sqrt{108} = 6\sqrt{3}$ m/s

Q. 3. (i)



$F_r = 20(4 - x - 1)$
$= 60 - 20x$

$F_l = 20(4 + x - 1) = 60 + 20x$

$F = F_r - F_l$
$= 60 - 20x - 60 - 20x$
$= -40x$

$F = ma \Rightarrow -40x = 5a \Rightarrow a = -8x$

It will perform SHM with $\omega = \sqrt{8}$. When it is released, $x = 3$, therefore $A = 3$.

(ii) Maximum speed $= \omega A = \sqrt{8}\,(3)$
$= 3\sqrt{8}$ m/s.

(iii) We want to find x when $v = \sqrt{8}$ m/s.
$v^2 = \omega^2(A^2 - x^2)$
$\Rightarrow 8 = 8(9 - x^2)$
$\Rightarrow 9 - x^2 = 1$
$\Rightarrow x = \sqrt{8}$ m

It starts from an extreme point.
$\therefore x = A \cos \omega t \Rightarrow x = 3 \cos \sqrt{8}t$

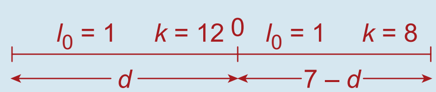
(iv) To find t when $x = \sqrt{8}$
$\Rightarrow \sqrt{8} = 3 \cos \sqrt{8}t$

$\Rightarrow \cos \sqrt{8}t = \dfrac{\sqrt{8}}{3} = \dfrac{2.828}{3} = 0.9427$

$\Rightarrow \sqrt{8}t = \cos^{-1}(0.9427) = 0.34$

$\Rightarrow t = \dfrac{0.34}{\sqrt{8}} = 0.12$ s

Q. 4. (i) Let 0 be the position of equilibrium



$F_r = k(l - l_0)$
$= 8(7 - d - 1)$
$= 48 - 8d$

$F_l = k(l - l_0)$
$= 12(d - 1)$
$= 12d - 12$