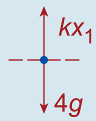
Q. 4. **For the 4 kg mass**

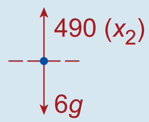


For equilibrium ↑ = ↓

$\Rightarrow kx_1 = 4g$ where x_1 = extension

$\Rightarrow k = \dfrac{4g}{0.08} \Rightarrow k = 490$ N/m

For the combined (6 kg) mass



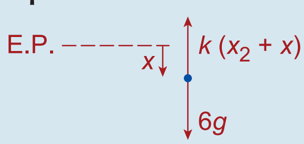
For equilibrium ↑ = ↓

$\Rightarrow 490\, x_2 = 6g$ ①

$\Rightarrow x_2 = 0.12$

Now, the Amplitude, A is $x_2 - x_1 = 0.04$ m

Combined mass at displacement x from equilibrium:



NZL: $\Sigma F = ma$

$\Rightarrow 6g - k(x_2 + x) = 6a$

But $6g = 490x_2$ from ①

(i) $\Rightarrow 490x_2 - 490(x_2 + x) = 6a$

$\Rightarrow a = -\dfrac{245}{3}x$

So the motion is Simple Harmonic with

$\omega = \sqrt{\dfrac{245}{3}}$

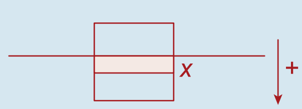
(ii) $\Rightarrow T = \dfrac{2\pi}{\omega}$

$= 2\pi\sqrt{\dfrac{3}{245}}$ **OR** $2\pi\sqrt{\dfrac{3}{25g}}$ s

(iii) $V_{MAX} = \omega A = \sqrt{\dfrac{245}{3}}(0.04)$

$= \dfrac{7\sqrt{15}}{75}$ **OR** $\dfrac{1}{5}\sqrt{\dfrac{9}{3}}$ m/s

Q. 5. (i) Mass $= V_P = h^3(1000\, s) = 1000\, h^3 s$



When it is depressed a distance x, the extra bouyancy B' is given by (note the downward is positive)

$B' = -$weight of displaced liquid

$= -V_{Pg} = (h^2x)(1000)g = -1000\, h^2xg$

$F = ma$

$\Rightarrow -1000\, h^2xg = (1000\, h^3s)\, a$

$\Rightarrow a = -\dfrac{x}{hs}$

$\therefore$ It will perform SHM with $\omega = \sqrt{\dfrac{g}{hs}}$.

The periodic time $= \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{hs}{g}}$

(ii) In this case $B' = -(h^2x)\,(1000\text{ k})g$
$= 1000\, h^2xkg$

$F = ma$

$\Rightarrow -1000\, h^2xkg = 1000\, h^3sa$

$\Rightarrow a = -\dfrac{gk}{hs}x$

$\therefore$ SHM with $\omega = \sqrt{\dfrac{gk}{hs}}$

Periodic time $= \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{hs}{gk}}$.

Q. 6. (i) As before, 0.6 of its height will be submerged, i.e. (0.6)(80) = 48 cm.

(ii) Originally, its displacement from equilibrium is 2 cm = 0.02 m.

Therefore $A = 0.02 = \dfrac{1}{50}$.

Mass $= V_p = [(0.8)(0.5)(0.2)]\,(600) = 48$ kg

When it is displaced a distance x m below the water, the extra buoyancy B′ is given by:

B′ = weight of liquid displaced

$= V_{pg} = (0.5)(0.2)(x)(1000)(9.8) = -980x$

$F = ma$

$\Rightarrow -980x = 48a$

$\Rightarrow a = -\dfrac{980}{48}x = -\dfrac{245}{12}x$

This is SHM with $\omega = \sqrt{\dfrac{245}{12}}$

Maximum acceleration $= \omega^2 A$

$= \dfrac{245}{12} \times \dfrac{1}{50} = \dfrac{49}{120}$ m/s^2