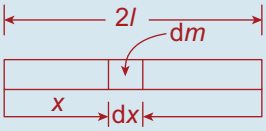
Chapter 14 Exercise 14A

Q. 1.



(i) $\rho = \frac{m}{2l}$, $dm = \rho\, dx$

$dI = x^2\, dm$

$\Rightarrow I = \rho\int x^2\, dx$

$\Rightarrow I = \frac{m}{3(2l)}\left[x^3\right]_0^{2l}$

$\Rightarrow I = \frac{m8l^3}{6l}$

$\Rightarrow I = \frac{4ml^2}{3}$ **QED**

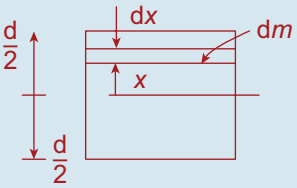
(ii) $mk^2 = I$

$\Rightarrow k = \sqrt{\frac{I}{m}}$

$= \sqrt{\frac{4\cancel{m}l^2}{3\cancel{m}}}$

$= \sqrt{\frac{4l^2}{3}}$ **OR** $\frac{2l}{\sqrt{3}}$

Q. 2.



$\rho = \frac{m}{d^2}$

(i) $dI = x^2\, dm$, $dm = \rho\, dA$

$\Rightarrow I = \frac{m}{d}\int_{-\frac{d}{2}}^{\frac{d}{2}} x^2\, dx$ $= \rho d\, dx$

$= \frac{m}{d^2} d dx$

$= \frac{m}{3d}\left[x^3\right]_{\frac{-d}{2}}^{\frac{d}{2}}$ $= \frac{m}{d} dx$

$= \frac{m}{3d}\left[\frac{d^3}{8} + \frac{d^3}{8}\right]$

$= \frac{2md^3}{24d}$

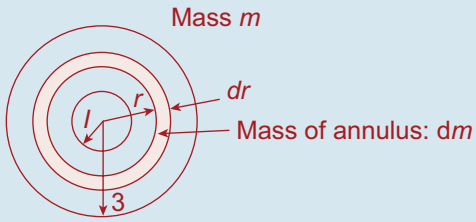
$I = \frac{md^2}{12}$ **QED**

(ii) $k = \sqrt{\frac{I}{m}}$

$= \sqrt{\frac{\cancel{m}d^2}{12\cancel{m}}}$

$\Rightarrow k = \frac{d}{2\sqrt{3}}$

Q. 3.



$\rho = \frac{\text{Mass}}{\text{Area}}$

$= \frac{m}{\pi(3^2 - 1^2)}$

$= \frac{m}{8\pi}$

$dm = \rho\, dA$

$= \frac{m}{8\cancel{\pi}} 2\cancel{\pi} r\, dr$

$= \frac{mr}{4} dr$

So, $dI = r^2\, dm$

$\Rightarrow dI = \frac{m}{4} r^3\, dr$

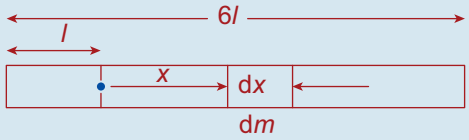
$\Rightarrow I = \frac{m}{4}\int_1^3 r^3\, dr$

$= \frac{m}{16}\left[r^4\right]_1^3$

$= \frac{m}{16}[81 - 1]$

$\Rightarrow I = 5m$ **QED**

Q. 4.



$dI = x^2\, dm$ $\rho = \frac{m}{6l}$

But $dm = \rho\, dx$

$= \frac{m}{6l} dx$

$\therefore I = \frac{m}{6l}\int_{-l}^{5l} x^2\, dx$

$= \frac{m}{18l}\left[x^3\right]_{-l}^{5l}$

$= \frac{m}{18l}[125l^3 + l^3]$

$= 7ml^2$