

Gain in K.E. = Loss in P.E.

$$\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$$

$$\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}(mr^2)\left(\frac{v^2}{r^2}\right) = mg(s \sin 30^\circ)$$

...assuming it rolls a distance s downhill

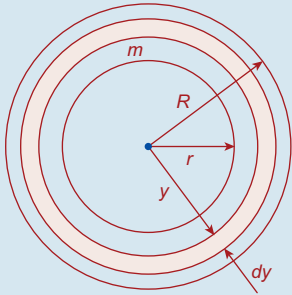
$$\Rightarrow \frac{1}{2}v^2 + \frac{1}{2}v^2 = \frac{gs}{2}$$

$$\Rightarrow v^2 = \frac{gs}{2}$$

$$v^2 = u^2 + 2as$$

$$\Rightarrow a = \frac{v^2 - u^2}{2s} = \frac{\frac{gs}{2} - 0}{2s} = \frac{1}{4}g$$

Q. 8.



$$\rho = \frac{m}{\pi(R^2 - r^2)}$$

$$dI = y^2 dm$$

$$= \rho y^2 dA$$

$$= 2\pi\rho y^2(y) dy$$

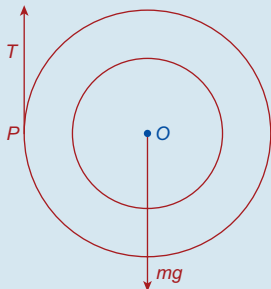
$$\Rightarrow I = \frac{2\pi\rho}{\pi(R^2 - r^2)} \int_r^R y^3 dy$$

$$= \frac{2m}{4(R^2 - r^2)} \left[y^4 \right]_r^R$$

$$= \frac{m}{2(R^2 - r^2)} (R^4 - r^4)$$

$$(i) = \frac{m(R^2 - r^2)(R^2 + r^2)}{2(R^2 - r^2)}$$

$$\Rightarrow I = \frac{m}{2}(R^2 + r^2) \quad \text{QED}$$



$$\Rightarrow I_P = \frac{m}{2}(R^2 + r^2) + mR^2$$

$$\Rightarrow I_P = \frac{m}{2}(3R^2 + r^2)$$

$$\text{NZL: } \Sigma L = I\ddot{\theta}$$

About p: ↙

$$mgR = \frac{m}{2}(3R^2 + r^2) \ddot{\theta}$$

(ii) Gain in K.E. = Loss in P.E.

$$\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$$

$$\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}\left[\frac{1}{2}m(3R^2 + r^2)\right]\left(\frac{v^2}{R^2}\right) = mgh$$

...multiply by $\frac{4R^2}{m}$

$$\Rightarrow 2v^2R^2 + (R^2 + r^2)v^2 = 4R^2gh$$

$$\Rightarrow v^2(2R^2 + R^2 + r^2) = 4R^2gh$$

$$\Rightarrow v^2(3R^2 + r^2) = 4R^2gh$$

$$\Rightarrow v^2 = \frac{4R^2gh}{3R^2 + r^2}$$

$$v^2 = u^2 + 2as$$

$$\Rightarrow a = \frac{v^2 - u^2}{2s}$$

$$= \frac{4R^2gh}{3R^2 + r^2} - 0$$

$$= \frac{2R^2g}{3R^2 + r^2}$$

(iii) $F = ma$

$$\Rightarrow mg - T = m\left(\frac{2R^2g}{3R^2 + r^2}\right)$$

$$\Rightarrow T = mg\left(1 - \frac{2R^2}{3R^2 + r^2}\right)$$

$$= mg\left(\frac{3R^2 + r^2 - 2R^2}{3R^2 + r^2}\right)$$

$$= mg\left(\frac{R^2 + r^2}{3R^2 + r^2}\right)$$

Exercise 14D

Q. 1. Rod: $I = \frac{4ml^2}{3}$ Length: $2l$

mass: m

For the Compound Pendulum:

$$T = 2\pi\sqrt{\frac{I}{mgh}}$$

Here $h = l$:

$$\therefore T = 2\pi\sqrt{\frac{4ml^2}{3mgl}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{4l}{3g}}$$