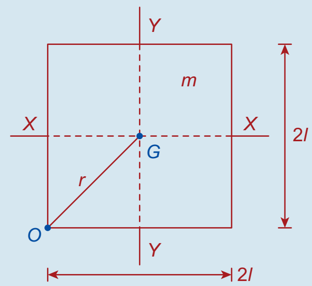
Q. 11. (i) Standard Proof

(ii)



$I_{XX} = I_{YY}$

$= \frac{m}{3} l^2$

Perpendicular Axes:

$I_G = I_{xx} + I_{yy}$

$I_G = \frac{2ml^2}{3}$

Parallel Axes:

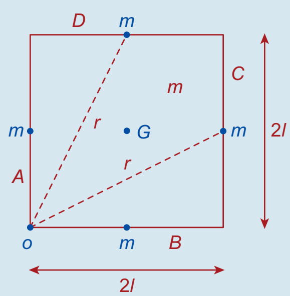
$I_O = I_G + mr^2$

where $r = \sqrt{2}l$

$\Rightarrow I_O = \frac{2ml^2}{3} + m(\sqrt{2}l)^2$

$\Rightarrow I_O = \frac{8ml^2}{3}$

(iii)



Lamina:

From (ii) $I_O = \frac{8ml^2}{3}$

Rods *A* and *B*:

$I_O = \frac{4}{3} ml^2$

Rods *C* and *D*:

$r = \sqrt{4l^2 + l^2}$

$\Rightarrow r = \sqrt{5}l$

$I_O = I_{\text{Mid point}} + mr^2$

$= \frac{ml^2}{3} + m(\sqrt{5}l)^2$

$\Rightarrow I_O = \frac{16ml^2}{3}$

So $I_{\text{Total}} = \frac{8ml^2}{3} + \frac{2(4ml^2)}{3} + \frac{2(16ml^2)}{3}$

$\Rightarrow I_{\text{Total}} = 16ml^2$ **QED**

(iv) **Note:** By symmetry, centre of gravity of system is at G

$\therefore h = r$ in part(ii)

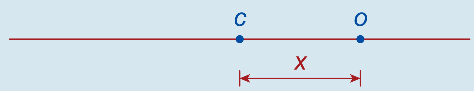
$\therefore T = 2\pi\sqrt{\frac{I}{mgh}}$

$T = 2\pi\sqrt{\frac{16ml^2}{5m\, g\sqrt{2}l}}$

$\Rightarrow T = 2\pi\sqrt{\frac{8\sqrt{2}l}{5g}}$ **QED**

Exercise 14E

Q. 1.



Length $2l$

mass m

$I_C = \frac{ml^2}{3}$ $\quad I_O = I_C + mx^2$, (| |Axes)

$I_O = \frac{ml^2}{3} + mx^2$

$= \frac{m(l^2 + 3x^2)}{3}$

$T = 2\pi\sqrt{\frac{I}{mgh}}$

$T = 2\pi\sqrt{\frac{m(l^2 + 3x^2)}{3mgx}} = 2\pi\left[\frac{l^2x^{-1}}{3g} + \frac{x}{g}\right]^{\frac{1}{2}}$

$\frac{dT}{dx} = \cancel{2}\pi\frac{1}{\cancel{2}}\left[\frac{l^2x^{-1}}{3g} + \frac{x}{g}\right]^{-\frac{1}{2}}\left[-\frac{l^2}{3gx^2} + \frac{1}{g}\right]$

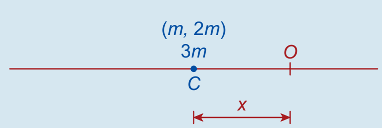
$\frac{dT}{dx} = \pi\frac{\left[\frac{1}{g} - \frac{l^2}{3gx^2}\right]}{\left[\frac{l^2}{3gx} + \frac{x}{g}\right]^{\frac{1}{2}}} = 0$ for minimum T

$\therefore \frac{l^2}{3gx^2} = \frac{1}{g}$

$\Rightarrow x^2 = \frac{l^2}{3}$

$\Rightarrow x = \frac{l}{\sqrt{3}}$ For minimum T

Q. 2.



Pt Mass: $2m$

$I_O = 2mx^2$

Rod: m

$I_C = \frac{ml^2}{3}$