||Axes:

$$I_O = I_C + mx^2$$

$$= \frac{ml^2}{3} + mx^2$$

$$I_{Total} = I_{Mass} + I_{Rod}$$

$$= 2mx^2 + \frac{ml^2}{3} + mx^2$$

$$= \frac{9mx^2 + ml^2}{3}$$

$$\left[T = 2\pi\sqrt{\frac{I}{mgh}}\right]$$

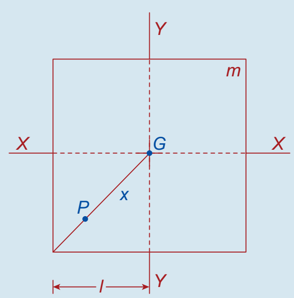
$$\Rightarrow T = 2\pi\sqrt{\frac{9mx^2 + ml^2}{3(3m)gx}}$$

$$\Rightarrow T = 2\pi\left[\frac{x}{g} + \frac{l^2}{9gx}\right]^{\frac{1}{2}}, \quad \text{For Min } T, \frac{dT}{dx} = 0$$

$$\frac{dT}{dx} = \frac{2\pi}{2}\left[\frac{x}{g} + \frac{l^2}{9gx}\right]^{\frac{-1}{2}}\left[\frac{1}{g} - \frac{l^2}{9gx^2}\right]$$

$$\therefore \frac{1}{g} = \frac{l^2}{9gx^2} \qquad \text{For Min } T \Rightarrow x = \frac{l}{3}$$

Q. 3. Square:



$$I_{xx} = I_{yy}$$

$$= \frac{ml^2}{3}$$

⊥ Axes:

$$I_G = ZI_{xx}$$

$$= \frac{2ml^2}{3}$$

||Axes:

$$I_p = I_G + mx^2$$

$$= \frac{2ml^2 + 3mx^2}{3}$$

$$\left[T = 2\pi\sqrt{\frac{I}{mgh}}\right]$$

$$= 2\pi\sqrt{\frac{2ml^2 + 3mx^2}{3mgx}}$$

$$= 2\pi\left[\frac{2l^2}{3gx} + \frac{x}{g}\right]^{\frac{1}{2}}$$

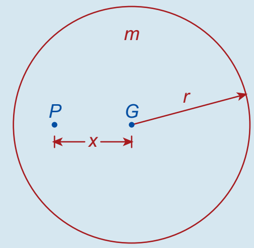
$$\frac{dT}{dx} = \frac{2\pi}{2}\left[\frac{2l^2}{3gx} + \frac{x}{g}\right]^{\frac{-1}{2}}\left[\frac{-2l^2}{3gx^2} + \frac{1}{g}\right]$$

$$\frac{dT}{dx} = 0 \qquad \text{For minimum } T$$

$$\therefore \frac{2l^2}{3gx^2} = \frac{1}{g}$$

$$x = \sqrt{\frac{2}{3}}l \qquad \textbf{QED}$$

Q. 4.



Disc:

$$I_G = \frac{mr^2}{2}$$

||Axes:

$$I_p = \frac{mr^2}{2} + mx^2$$

$$= \frac{mr^2 + 2mx^2}{2}$$

$$\left[T = 2\pi\sqrt{\frac{I}{mgh}}\right]$$

$$\Rightarrow T = 2\pi\sqrt{\frac{mr^2 + 2mx^2}{2mgx}} \quad \ldots ①$$

If $x = \frac{r}{2}$, we get

$$T = 2\pi\sqrt{\frac{mr^2 + \frac{mr^2}{2}}{mgr}}$$

$$= 2\pi\sqrt{\frac{3r}{2g}}$$

For minimum T, $\frac{dt}{dx} = 0$

From ①

$$T = 2\pi\left[\frac{r^2}{2gx} + \frac{x}{g}\right]^{\frac{1}{2}} \quad \ldots ②$$

$$\frac{dT}{dx} = \frac{2\pi}{2}\left[\frac{r^2}{2gx} + \frac{x}{g}\right]^{-\frac{1}{2}}\left[-\frac{r^2}{2gx^2} + \frac{1}{g}\right]$$

$$\frac{dT}{dx} = 0$$

$$\Rightarrow \frac{r^2}{2gx^2} + \frac{1}{g} \Rightarrow x = \frac{r}{\sqrt{2}} \qquad \textbf{QED}$$

From ①

$$T = 2\pi\sqrt{\frac{mr^2 + \frac{2mr^2}{2}}{\frac{2mgr}{\sqrt{2}}}}$$

$$= 2\pi\sqrt{\frac{r}{g}\sqrt{2}} < 2\pi\sqrt{\frac{3r}{2g}}$$

since $\sqrt{2} < \frac{3}{2}$