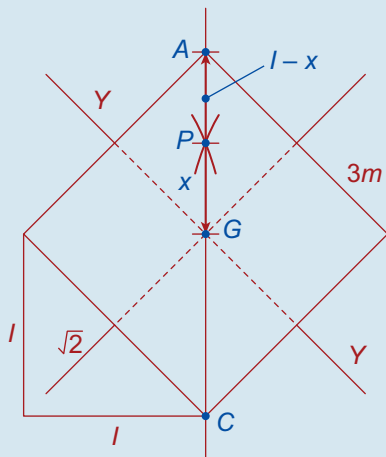


**Q. 5. Square Lamina:**



$$(i) I_{YY} = \frac{3m}{3} \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{m}{2}$$

$$I_G = 2I_{YY} \text{ (}\perp\text{ Axes)}$$

$$\Rightarrow I_G = m$$

|| Axes:

$$I_P = I_G + 3mx^2$$

$$\Rightarrow I_P = m + 3mx^2$$

**Pt. Mass:** A:  $I_P = m(1-x)^2$

C:  $I_P = m(1+x)^2$

$$I_{Total} = I_{Lamina} + I_{Pt. Mass}$$

$$= m + 3mx^2 + m(1-2x+x^2)$$

$$+ m(1+2x+x^2)$$

$$= 3m + 5mx^2$$

(ii)  $T = 2\pi\sqrt{\frac{I}{mgh}}$ , By symmetry centre of gravity of system is at G.

$$\Rightarrow T^2 = 4\pi^2 \left[ \frac{m(3+5x^2)}{5mgx} \right]$$

$$\Rightarrow T^2 = \frac{4\pi^2}{5g} \left[ \frac{3+5x^2}{x} \right] \quad \text{QED}$$

(iii)  $\therefore \frac{d(T)^2}{dx}$

$$= \frac{4\pi^2}{5g} \left\{ \frac{1}{x^2} [x(10x) - (3+5x^2)(1)] \right\}$$

$$= \frac{4\pi^2}{5gx^2} [5x^2 - 3]$$

For Minimum T,  $\frac{dT^2}{dx} = 0$

$$\therefore 5x^2 = 3$$

$$\Rightarrow x = \sqrt{\frac{3}{5}}$$

**Q. 6. (a) Standard Proof**

**(b) Standard Proof**

**(c) Rod:**  $I_A = \frac{4}{3} ma^2$

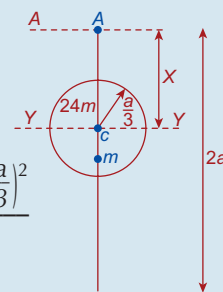
**Disc:**  $I_{AA} = \frac{24m \left(\frac{a}{3}\right)^2}{4}$

$$I_{YY} = \frac{2ma^2}{3}$$

|| Axes:  $I_{AA} = \frac{2ma^2}{3} + 24mx^2$

$$\therefore I_{Total} = I_{Rod} + I_{Disc}$$

$$= \frac{4}{3}ma^2 + \frac{2ma^2}{3} + 24mx^2$$



(i)  $\Rightarrow I_{TOTAL} = 2m(a^2 + 12x^2) \quad \text{QED}$

(ii) **Note:** In this question the plane of the disc is perpendicular to the plane in which the rod moves, i.e. as shown in diagram the disc moves Into the page.

Now,  $T = 2\pi\sqrt{\frac{I}{mgh}}$

$\therefore$  We need to find  $h$ , (the position of centre of gravity)

**Taking moments about A:**

$$m(a) + 24mx = 25mh$$

$$\Rightarrow h = \frac{a + 24x}{25}$$

$$\text{So } T = 2\pi\sqrt{\frac{2m(a^2 + 12x^2)(25)}{25m(g)(a + 24x)}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{2(a^2 + 12x^2)}{g(a + 24x)}}$$

Now, Minimum T occurs

for  $\frac{(a^2 + 12x^2)}{(a + 24x)}$  minimised

$$\therefore \frac{1}{(a + 24x)^2} [(a + 24x)(24x) - (a^2 + 12x^2)(24)]$$

$$\Rightarrow (a + 24x)(24x) = (a^2 + 12x^2)(24)$$

$$\Rightarrow a + 24x^2 = a^2 + 12x^2$$

$$12x^2 + ax - a^2 = 0$$

$$\Rightarrow (3x + a)(4x - a) = 0$$

$$\Rightarrow x = \frac{a}{4} \quad \text{For minimum T} \quad \text{QED}$$